# Skin effect in strongly inhomogeneous laser plasmas with weakly anisotropic temperature distribution

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The absorption coefficient of an ultrashort, high-intensity *S*-polarized laser pulse is calculated for plasmas with weakly anisotropic electron energy distribution functions and high gradients of electron density. In the limiting cases of normal and anomalous skin depth effects, the plasma kinetic equation coupled to the Maxwell equations can be solved analytically for different relations between the laser wavelength, the electron density gradient scale length, and the skin depth. For anisotropic electron distribution functions, we obtained an increase of the laser absorption coefficient for transverse to longitudinal electron temperature ratio greater than one. [S1063-651X(98)00508-X]

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#### I. INTRODUCTION

Since the development of the technique of chirped pulse amplification to build compact solid-state lasers [1], investigations of the interaction processes of ultrashort ( $\tau_p$ ) <0.1 ps), high-intensity ( $I\lambda^2 > 10^{16}$  W/cm<sup>2</sup>  $\mu$ m<sup>2</sup>) laser pulses with solid targets have been performed intensively [2]. During the interaction, a dense  $(n_e > 10^{22} \text{ cm}^{-3})$ , hot  $(\approx keV)$  plasma is produced by laser absorption at the surface of the target; the laser deposition thickness is comparable to the material skin depth  $(l_s \approx 10 \text{ nm})$ . In previous works [3,4], the theory of the skin effect for such plasmas was developed within the approximation of an abrupt boundary between the plasma and vacuum. However, Yang and his collaborators showed recently in numerical simulations the considerable influence of plasma inhomogeneity on laser absorption [5]. The plasma inhomogeneity always exists in an experiment because of the limited intensity contrast ratio between the main pulse and its amplified spontaneous emission background. Then, the density inhomogeneity scale length is  $L < \lambda$ , where  $\lambda$  is the wavelength of laser radiation. As an example, it has been early recognized experimentally [6] that the presence of a weak prepulse boosts the x-ray conversion efficiency from solid target because the laser interacts not with the surface of the solid material but with a preformed plasma.

The theory of the skin effect in one-dimensional inhomogeneous plasma with an isotropic temperature distribution was formulated for the first time in Ref. [7]. The present work generalizes the theory of skin effect in one-dimensional inhomogeneous plasma to the case of an anisotropic electron distribution function and to the case of oblique incidence of an *S*-polarized laser wave. The magnetic field of the wave, that was not taken into account in Ref. [7], will play an important role in the case of a plasma with an energy distribution anisotropy and oblique incidence. We consider the same regime of laser-plasma interaction as in Ref. [8] where nonlinear effects play a negligible role. Accordingly, calculation of the skin effect will be performed within the framework of the perturbation theory. This limits the electric field strength E in the plasma by the condition  $v_E = eE/m\omega < v_T$  $=\sqrt{T_e/m}$ , where  $v_E$  is the oscillation velocity of an electron in the laser field,  $v_T$  is the thermal velocity, and  $\omega$  the laser frequency. Corresponding laser intensities  $I\lambda^2$  are below  $10^{17}$  W/cm<sup>2</sup>  $\mu$ m<sup>2</sup>. In this paper, we consider mainly the "collisionless" mechanisms of absorption of a laser pulse in a strongly inhomogeneous plasma. These mechanisms are the anomalous skin effect (ASE) [3,4], when the effective collision frequency  $\nu$  is less than laser frequency  $\omega$  and the ratio of thermal velocity to skin length  $v_T/l_s$ ;  $\nu \ll \omega \ll v_T/l_s$ ; and the sheath inverse bremsstrahlung (SIB) [5] when  $\nu$  $\ll v_T/l_s \ll \omega$ . The condition on laser intensity for linear ASE to prevail [3] is  $v_E < v_T$  or  $(I/10^{12} \text{ W/cm}^2) \le 60(A/Z) [\pi/$  $(1 fs)]^{2} [\lambda/(1 \mu m)]^{-6}$  where Z is the average charge of the plasma and the condition for the pulse duration is L  $< v_T/\omega$  or  $\tau(fs) < 30(A/z)^{1/2}\lambda(\mu m)$ . From these conditions, we see that ASE is important for  $I \le 10^{17}$  W/cm<sup>2</sup>,  $\tau \le 40$  fs, and  $\lambda = 1 \mu m$ . For SIB, the conditions on laser intensity can be written as [5]  $I(10^{18} \text{ W/cm}^2) \le (Z/10)^{7/3} (n_i/6 \times 10^{22} \text{ cm}^3)^{5/3} [\lambda/(1 \ \mu\text{m})]^{2/3}$  and  $\lambda \le 0.5 \ \mu\text{m}$ . To reach this regime, it is necessary to use much shorter laser wavelengths than for ASE but the pulse duration can be of the order of 100 fs.

We have a deformation of the electron distribution function during plasma heating by the laser pulse. In the spatial region close to the plasma-vacuum boundary the distribution function differs from the Maxwellian one by a deficit of slow particles [4]. In the case of ASE, the anisotropy of the laser heating in the skin layer can be explained by the fact that electrons with low longitudinal (parallel to the laser wave vector) velocities are accelerated in a direction parallel to the laser field so that their transverse energy can become much larger than their longitudinal energy [9]. For the laser intensities we consider, this anisotropy is not very large and has a

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weak dependence on time [4]. Deeper into the plasma, at a distance from the boundary much larger than the electron mean free path, the anisotropy in the electron distribution function is reduced because of the increasing role played by electron-ion collisions.

It is well known that plasmas with temperature anisotropy are unstable [10]. Gain estimations shows that, in our case, it is inversely proportional to the pulse duration for temperature anisotropy parameter  $\Delta = (T_{\perp} - T_{\parallel})/T_{\parallel} > 1$ . We conclude that for weak temperature anisotropy we can neglect this instability.

## **II. INITIAL SET OF EQUATIONS**

We consider the oblique incidence (with an angle  $\theta$  with respect to the normal  $O_z$  of the target plane) of an *S*-polarized laser wave on a nonrelativistic plasma that is inhomogeneous along the *z* axis. It is more convenient to perform calculations in the reference system moving with the velocity  $v/c = \sin \theta$  along the plasma surface in the positive direction of the *x* axis [11]. The incidence is normal in this reference frame. The field strengths are  $E_y = E_y^{(0)} \cos \theta$ ;  $B_x$  $= B^{(0)} \cos \theta$ ;  $\omega = \omega^{(0)} \cos \theta$ , where the index 0 stands for the values in the laboratory frame. The inhomogeneity of the plasma density in the direction of the *z* axis produces an ambipolar field  $E_z^{a(0)} = -\partial \varphi^{(0)}/\partial z$ , deriving from the potential  $\varphi^{(0)}$ , which acts on the plasma electrons from the side of the slowly moving ions. In the moving coordinate system, this field  $E_z^a = E_z^{a(0)}/\cos \theta = -\partial \varphi/\partial z$  causes an additional magnetic field  $B_y = \tan \theta E_z^{a(0)}$ . As a result, the kinetic equation for electrons has the following form:

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} + \left(\frac{eE^{a(0)}}{\cos\theta} + \frac{e}{c} v_x \tan\theta E^{a(0)}\right) \frac{\partial f}{\partial p_z} - \frac{e}{c} v_z \tan\theta E^{a(0)} \frac{\partial f}{\partial p_x} + v(f - f_M)$$
$$= \left\{ -eE_y^{(0)} \cos\theta \frac{\partial}{\partial p_y} - \frac{e}{c} B_x^{(0)} \cos\theta \left( v_z \frac{\partial}{\partial p_y} - v_y \frac{\partial}{\partial p_z} \right) \right\} f, \tag{1}$$

where *f* is the electron distribution function,  $f_M$  the equilibrium distribution function (for example a bi-Maxwellian), and  $\nu$  the electron-ion collision frequency (we consider a laser plasma with a mean ion charge  $Z \ge 1$  so that the collision integral giving  $\nu$  is well known).

Electromagnetic fields in the moving system satisfy the Maxwell equations:

$$\frac{\partial E_y}{\partial z} = \frac{1}{c} \frac{\partial B_x}{\partial t}, \quad \frac{\partial B_x}{\partial z} = \frac{4\pi}{c} j_y + \frac{1}{c} \frac{\partial E_y}{\partial t},$$
$$\frac{\partial^2 \varphi}{\partial z^2} = -4\pi e \left[ \int f d^3 p - \frac{n_i^{(0)}(z)}{\cos \theta} \right], \tag{2}$$

where  $n_i^{(0)}(z)$  is the profile of ion concentration in the laboratory system of coordinates. The function  $n_i^{(0)}(z)$  is assumed to be given and constant during the whole process of interaction, because the time scale of the hydrodynamic motion of the ions is much larger than the time duration  $\tau_p$  of the laser pulse.

By means of a generalization of the method developed in Ref. [7], we obtain the equation for the laser field  $E_y$ :

$$\frac{d^{2}E_{y}}{dz^{2}} + \frac{\omega^{2}}{c^{2}}E_{y} = -\frac{4\pi i\omega e^{2}}{mc^{2}}\left\{\int_{z}^{\infty}dz'\int_{0}^{p_{z}\max}dp_{z}W^{-1/2}e^{-\Phi(z';z^{*})}2\cosh\Phi(z;z^{*})[E_{y}(z') + \hat{B}_{1}(z')]f_{M}(z';p_{z})\right. \\
\left. + \int_{z}^{z}\max dz'\int_{p_{1}}^{p_{z}\max}W^{-1/2}e^{-\Phi(z;z^{*})}2[\cosh\Phi(z';z^{*})E_{y}(z') - \sinh\Phi(z';z^{*})\hat{B}_{1}(z')]f_{M}(z',p_{z}) \\
\left. + \int_{z}^{\infty}dz'\int_{p_{z}\max}dp_{z}W^{-1/2}e^{\Phi(z;z')}[E_{y}(z') + \hat{B}_{1}(z')]f_{M}(z';p_{z}) + \int_{z}^{z}\max dz'\int_{p_{z}\max}dp_{z}W^{-1/2}e^{-\Phi(z;z')} \\
\left. \times [E_{y}(z') - \hat{B}_{1}(z')]f_{M}(z';p_{z}) \right\},$$
(3)

where

$$W = \frac{2\cos^2\theta}{m} \left\{ \frac{p_z^2}{2m} + e \, \cos \, \theta[\,\varphi(z) - \varphi(z')\,] \right\},\,$$

$$\hat{B}_{1}(z) = \frac{ip_{z}\cos\theta}{m\omega} \frac{\partial E_{y}}{\partial z} \frac{T_{\perp} - T_{\parallel}}{T_{\parallel}}, \quad p_{1} = \{2em \cos^{3}\theta[\varphi(z') - \varphi(z)]\}^{1/2}.$$
$$\Phi(z_{1};z_{2}) = \int_{z_{1}}^{z_{2}} (-i\omega + \nu)X^{-1/2}(z,z'')dz'',$$
$$X(z,z') = \frac{2\cos\theta}{m} \left\{ \frac{p_{z}^{2}\cos\theta}{2m} + [e\varphi(z) - e\varphi(z')] \left[1 + \frac{p_{x}}{mc}\sin\theta\cos\theta\right] \right\}.$$

The last two terms in Eq. (3) describe the current of particles having energies in excess of the barrier  $e\varphi_{max}$ . The structure of Eq. (3) is too complex to get a general analytical solution. Therefore, we will analyze it in a number of limiting cases. First of all, consider a high potential barrier  $e\varphi_{max} \gg T_{\parallel}$  so that electrons cannot escape the plasma. Further simplification can be performed with some approximations that we will consider in the following parts of the paper. To investigate the general solution of Eq. (3) in various limiting cases, we will study the dependence of the absorption coefficient as a function of the scale of plasma inhomogeneity *L*. In Sec. III, we will consider the case of homogeneous plasmas with a sharp boundary when the scale length is less than a skin layer. In Sec. IV, we will consider the case of inhomogeneous plasmas with weak and strong dispersion.

## III. HOMOGENEOUS PLASMAS WITH SHARP PLASMA-VACUUM BOUNDARY

Analytical calculation of the integrals in Eq. (3) is possible for a restricted number of potentials  $\varphi(z)$ . First of all, consider the well investigated case of an abrupt plasma boundary [3–5]. Solution of this case by the Fourier transform method with  $T_{\parallel}=T_{\perp}$  was performed in [4,5]. In these works, the variation of the absorption coefficient  $\eta$  with the parameter  $\omega c/\omega_p v_{T_{\parallel}}$  has been calculated. The limit  $c/\omega_p \gg v_T/\omega$  corresponds to the high frequency normal skin effect, the so-called sheath inverse bremsstrahlung (SIB) [5], when the depth of the skin layer  $c/\omega_p$  is much greater than the scale of spatial dispersion  $v_T/\omega$ . The opposite case  $c/\omega_p \ll v_T/\omega$  corresponds to anomalous skin effect (ASE) investigated in [3,4]. Generalization of the results of [4] and [5] to the case of an anisotropic plasma with collisions gives an absorption coefficient of

$$\eta = \left(\frac{c}{8\pi} |E(-\infty)|^2\right)^{-1} \frac{1}{2} \operatorname{Re} \int_0^\infty dz \ \vec{E}^*(z) \vec{j}(z)$$
$$= 1 - \left|\frac{(\zeta \cos \theta - 1)}{(\zeta \cos \theta + 1)}\right|^2, \tag{4}$$

where

$$\zeta = \frac{2iv_{T_{\parallel}}}{\pi c} \int_0^\infty \frac{d\xi}{1 - (\xi^3/\alpha^2) \{Z(\xi) + \Delta[1/\xi + Z(\xi)]\}}, \quad (5)$$

and

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$$Z(\xi) = \exp[-(\xi + i\,\delta)^2] \left\{ i\sqrt{\pi} \operatorname{erfc}(\delta\xi) - 2\int_0^{\xi} \exp(t + i\,\delta\xi)^2 dt \right\}, \quad \text{with} \quad \delta = \nu/\omega$$

Figure 1 shows the variation of the laser absorption coefficient with the skin depth effect parameter  $(c \omega/\omega_p v_T)^2$  and anisotropy parameter  $\Delta = (T_{\perp} - T_{\parallel})/T_{\parallel}$ . In this case, laser absorption is caused by mainly Landau damping, as  $\delta = \nu/\omega = 0.01$ . It is evident from Fig. 1 that plasma anisotropy causes an increase of the absorption coefficient for  $T_{\perp} > T_{\parallel}$  [4]. Additionally, we can see that for  $\Delta = 0$ , our results are the same than the ones obtained in Ref. [5].

#### **IV. INHOMOGENEOUS PLASMAS**

Now we consider the case of a smoothly inhomogeneous plasma. One of the potentials that gives a possibility for an analytical calculation of the phases  $\Phi(z_1;z_2)$  is the linear potential,  $e\varphi \cos^2 \theta/T_{\parallel} = -z/L$ . It corresponds to an exponential electron density gradient with a scale length of inhomogeneity  $L_n = L/\cos \theta$ . In this case, the problem has a physical interpretation only for  $L \gg l_s$ , where  $l_s$  is the depth of the skin layer. Introducing the dimensionless coordinate  $\xi = z/L$ , we obtain the following form for Eq. (3) with the linear potential:

$$\frac{d^{2}E_{y}}{d\xi^{2}} + \kappa^{2}E_{y} = -\frac{iA}{\sqrt{\pi}} \int_{-\infty}^{+\infty} d\xi' \int_{-\infty}^{+\infty} dt \left\{ (e^{i\alpha\sqrt{|\xi'-\xi|}e^{-t}} + e^{i\alpha\sqrt{|\xi'-\xi|}e^{t}})E_{y}(\xi') + \frac{i}{2\alpha}\Delta \frac{\partial E_{y}}{\partial\xi'}\sqrt{|\xi'-\xi|}(e^{i\alpha\sqrt{|\xi'-\xi|}e^{-t}} + \operatorname{sgn}(\xi'-\xi)e^{i\alpha\sqrt{|\xi'-\xi|}e^{t}})[e^{t} - \operatorname{sgn}(\xi'-\xi)e^{-t}] \right\} e^{(|\xi'-\xi|/2)\operatorname{cosh} 2t} e^{(3\xi'-\xi)/2},$$
(6)

where

$$A = \frac{\omega \omega_p^2 L^3}{c^2 v_{T_{\parallel}}} \approx \frac{L^3}{l_{Sa}^3}, \quad \kappa^2 = \frac{\omega^2 L^2}{c^2},$$
$$\alpha = \frac{L\omega(1+iv/\omega)}{v_{T_{\parallel}}}, \quad \omega_p^2 = \frac{4\pi e^2 n^{(0)}}{m}.$$

The right side of Eq. (6) is the plasma current driven by the electric field of the laser wave. Nonlocal dependence of this current from the field is connected with spatial dispersion of the inhomogeneous plasmas.

#### A. The limit of sheath inverse bremsstrahlung

Here we consider the solution of Eq. (6) in the limiting case  $\alpha \ge 1$  (SIB) [12]. In this weak spatial dispersion case, Eq. (6) involves two considerably different scale lengths: the small scale  $\approx v_{T_{\parallel}}/\omega$  and the large one  $\sim L$ . So, the variation of density on the coordinate  $\xi$  can be taken into account adiabatically, and Eq. (6) is reduced to an ordinary differential equation for the field with an effective dielectric permitivity  $\varepsilon(q;\omega)$ :

$$\frac{d^2 E_y}{d\xi^2} + \kappa^2 E_y + \kappa^2 \{ \varepsilon [q(\omega,\xi);\omega] - 1 \} e^{\xi} E_y = 0.$$
(7)

The dielectric permitivity  $\varepsilon(q,\omega)$  can be obtained by taking the Fourier transform of Eq. (6) with respect to the small scale length  $v_T/\omega$ :

$$\varepsilon(q,\omega) \cong 1 - \frac{A}{\kappa^2 \alpha} + \frac{iA\sqrt{\pi}}{2\kappa^2} \frac{1}{\sqrt{|q|}} \exp\left\{-\frac{\alpha^2}{4|q|}\right\}.$$
(8)

The imaginary part of the dielectric permitivity is different from zero in the collisionless limit ( $\nu = 0$ ) because of Landau damping. Below, because of the small value of Im  $\varepsilon$  for  $\alpha \ge 1$ , we will calculate the absorption within the perturbation theory.

The expression of  $q(\xi; \omega)$  is obtained from the dispersion equation

$$-q^{2}+\kappa^{2}\left(1-\frac{Ae^{\xi}}{\kappa^{2}\alpha}\right)=0,$$
(9)

in which the smooth dependence on the reduced coordinate was taken into account.

The absorption coefficient  $\eta$  can be calculated as the ratio of the real part of the work performed by the field per unit of time per unit of plasma area to the Poynting vector of the wave incident on the plasma:

$$\eta = \frac{\int_{-\infty}^{\infty} d\xi \ E_{y}^{2}(\xi) (A \sqrt{\pi/2\kappa}) [1/\sqrt{|q(\xi)|}] \exp\{-\alpha^{2}/4|q(\xi)|\} e^{\xi}}{E_{y}^{2}(\xi \to -\infty)}.$$
(10)

A solution of Eq. (7) decreasing at  $\xi \rightarrow \infty$  can be found in the form of a superposition of the incident wave with the specified amplitude and the reflected wave. This gives for Eq. (10), using the saddle point method, the following expression:

$$\eta \approx \sqrt{2} \pi A^{5/4} \alpha^{1/4} \exp(-3 \alpha^{5/4} A^{-1/4}) \sinh(2 \pi \kappa) / \kappa.$$
(10a)

So, in the SIB regime, at  $0 < L < c/\omega_p$ , the absorption coefficient can be taken from Eq. (5) and at  $L > c/\omega_p$  from Eq. (10).

Because we have made the assumption  $\alpha \ge 1$ , we can see from Eq. (10a) that we have a decrease of the absorption coefficient for increasing L. We can conclude that in plasmas with weak dispersion, we are in the limit of SIB at any scale L. The absorption coefficient is maximum for sharp boundaries and falls down for increasing L.

Figure 2(a) shows the variation of the laser absorption coefficient  $\eta$  with the scale of plasma inhomogenity L for  $\omega_p^2/\omega^2 = 20$ ,  $v_T/c = 0.1$ , in the conditions of SIB. The dashed line is calculated with help of Eq. (10) and the results of PIC simulations of Yang and co-authors are denoted by  $\otimes$  symbols at the same parameters. We see a rather good agreement between the two sets of results for kL < 0.2 and some differ-

ences for larger kL that can be connected to the different electron density profiles used in the calculations.

#### B. The limit of anomalous skin effect

Now we will consider the case of strong spatial dispersion, opposite to that of the previous section, the so-called anomalous skin effect (ASE). In this case, the scale of spatial dispersion  $v_T/\omega$  exceeds the scale of inhomogeneity *L*, therefore in the general expression given in Eq. (3), the phases  $\Phi(z_1;z_2)$  are small and can be omitted.

In the case of the linear potential that we consider now, the parameter  $\alpha$  in Eq. (6) becomes small, and the later equation can be written in a simpler form:

$$\frac{d^2 E_y}{d\xi^2} + \kappa^2 E_y = -\frac{iA}{\sqrt{\pi}}$$

$$\times \int_{-\infty}^{+\infty} d\xi' K_0 \left(\frac{|\xi - \xi'|}{2}\right) e^{(\xi - \xi')/2} e^{\xi'} E_y(\xi').$$
(11)

The kernel of the integral in Eq. (11)  $K_0(\xi - \xi')$  is such that it cuts integration over  $d\xi'$  at  $\xi - \xi' \approx 1$  for  $\xi > \xi'$  and at  $\xi' - \xi \approx 1/\alpha^2$  for  $\xi < \xi'$ . This conclusion follows from the

$$\frac{d^3 E_y}{d\mu^3} + \kappa^2 \frac{\partial E_y}{\partial \mu} - \frac{iA}{\sqrt{\pi}} C e^{\mu} E_y(\mu) = 0, \quad \mu = \xi - 1, \quad (12)$$

where C is a numerical constant of the order unity.

After the change of variable  $e^{\mu} = x$ , Eq. (12) is reduced to the standard form of the equation for the generalized hypergeometric function  $_{0}F_{2}$  [14]. Using the asymptotical behavior of  $_{0}F_{2}$  at large values of modulus of the argument [14], we can find the following expression for the amplitude reflection coefficient *R*:

$$R = \frac{1 + (\omega L/c) \left[ -\pi/2 + i (\ln \pi A/\sqrt{2} + \tilde{\gamma}C) \right]}{1 - (\omega L/c) \left[ -\pi/2 + i (\ln \pi A/\sqrt{2} + \tilde{\gamma}\widetilde{C}) \right]}$$
$$\approx 1 + 2 \frac{\omega L}{c} \left[ -\frac{\pi}{2} + i \left( \ln \frac{\pi A}{\sqrt{2}} + \tilde{\gamma}\widetilde{C} \right) \right], \qquad (13)$$

where  $\tilde{C} = 0.577$  is the Euler constant.

Similar calculations can be done for a potential  $\varphi(z)$  of the form  $-e\varphi(z)/T_{\parallel} = \ln z/L = \ln \xi$ , corresponding to a linear variation of the electron plasma density. Introducing a new variable  $x = \ln \xi$  and approximating the kernel, we again come to an equation of the hypergeometric type. In spite of the other expression of  $E_y(\xi)$ , the absorption coefficient is again proportional to the scale of the plasma inhomogeneity.

We have plotted  $\eta(L)$  in Fig. 2(b), denoted by a dotted line, which was found with the help of Eq. (13) for  $\omega_p^2/\omega^2 = 60$ ,  $v_T/c = 0.1$  to compare it with the results of [5]



FIG. 1. Variation of the absorption coefficient with the skin depth parameter  $(\omega c/\omega_p v_T)^2$  for two values of the anisotropy parameter  $\Delta = T_{\perp}/T_{\parallel} - 1$  and  $\theta = 0$ ,  $\nu/\omega = 0.01$ .



FIG. 2. Variation of the absorption coefficient as a function of the scale length of plasma inhomogeneity for an isotropic ( $\Delta = 0$ ) and anisotropic ( $\Delta = 0.5$ ) distribution. (a) SIB regime  $v_T/c = 0.1$ ,  $\omega_p^2/\omega^2 = 20$ . The dashed line shows our results. The encircled crosses ( $\otimes$ ) are the results of PIC simulations by Yang *et al.* (b) ASE regime  $v_T/c = 0.1$ ,  $\omega_p^2/\omega^2 = 60$ . The dotted line and the solid line show our results, the crosses ( $\times$ ) are the results of PIC simulations by Yang *et al.* 

(with the same physical parameters) denoted by symbols ×. Since  $c \omega/v_T \omega_p = 1.3$ , we are in the intermediate region between ASE and SIB. Even in this case, where our analytical formula is not so good, we can see that we have rather good agreement between the analytical and the numerical results. The comparison with the experimental results of Ref. [13], where the laser plasma parameters were close to our region of ASE (0.1 < kL < 0.3,  $I < 10^{18}$  W/cm<sup>2</sup>) show a rather good agreement with our calculations since  $\eta_{expt} = \eta_{calc} \sim 0.07$ .

We now consider the solution of the field equation, taking

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into account the temperature anisotropy. At first, we consider the case of  $[(T_{\perp} - T_{\parallel})/T_{\parallel}] 1/\alpha \ge 1$ . Estimation of the kernel of the integral equation (6) in a manner similar to what was done in Sec. III gives the following approximation for Eq. (6):

$$\frac{d^2 E_y}{d\xi^2} + \kappa^2 E_y = \frac{AC}{\alpha \sqrt{\pi}} \frac{T_\perp - T_\parallel}{T_\parallel} \int_{\xi}^{\infty} \frac{\partial E_y}{\partial \xi'} e^{\xi'} d\xi', \quad (14)$$

where C is a numeric constant of the order unity.

The solution of Eq. (14) is again a generalized hypergeometric function. The condition of a negligible electric field in the bulk of the target gives the relation between amplitude of the incident and reflected waves. As a result, the reflection coefficient is

$$R \approx e^{-3\pi\kappa} \left( \frac{AC}{\sqrt{\pi\alpha}} \frac{T_{\perp} - T_{\parallel}}{T_{\parallel}} \right)^{3i\kappa}$$
$$\approx 1 - 3\pi\kappa + 3i\kappa \ln \frac{AC}{\sqrt{\pi\alpha}} \frac{T_{\perp} - T_{\parallel}}{T_{\parallel}}, \quad T_{\perp} > T_{\parallel}. \quad (15)$$

In the opposite case of  $(T_{\perp} - T_{\parallel})/\alpha T_{\parallel} \ll 1$ , the anisotropic correction to the absorption coefficient can be estimated with perturbation theory:

$$\Delta \eta = \frac{A(T_{\perp} - T_{\parallel})}{\sqrt{\pi}\kappa\alpha T_{\parallel}} \cdot \frac{\int_{-\infty}^{+\infty} d\xi \int_{\xi}^{\infty} d\xi' \operatorname{Im} \frac{\partial E_{y}}{\partial\xi'} \operatorname{Re} E_{y}(\xi) \operatorname{erfc}(\sqrt{\xi' - \xi}) \exp(2\xi' - \xi)}{[\operatorname{Re} E_{y}(\xi = -\infty)]^{2}},$$
(16)

where  $E_y$  is determined by Eq. (14). Estimations for the sign of  $\Delta \eta$  give  $\Delta \eta > 0$  for  $T_{\perp} > T_{\parallel}$ , and  $\Delta \eta < 0$  for  $T_{\perp} < T_{\parallel}$ .

Thus, in the limit of great anisotropy,  $T_{\perp} > T_{\parallel}$ , the absorption coefficient increases by a factor of three [in the case of weak anisotropy, calculated from Eq. (16), the difference in absorption is not so much; see the solid line in Fig. 2(b)]. If  $T_{\perp} < T_{\parallel}$ , the corresponding analysis gives a decrease of the absorption coefficient as compared with the isotropic case.

## V. CONCLUSIONS

In the present work, we have derived an integrodifferential equation for the calculation of the electromagnetic field in a plasma, taking into account temperature anisotropy and one-dimensional spatial inhomogeneity. In the limiting cases of normal and anomalous skin effects, the equation can be simplified. This allows one to find its solutions in analytical form. A new feature of our calculations is the accounting of the magnetic field of the wave in the kinetic equations. In the case of an anisotropic electron distribution function, the important effect is the increase of the laser absorption coefficient  $\eta$  for increasing transverse to longitudinal electron temperature ratios. The functional dependence of the absorption coefficient  $\eta(L)$  with the electron density gradient L substantially depends on the regime of the skin effect ( $\alpha \ll 1$  for the ASE, and  $\alpha \ge 1$  for the SIB). If  $\alpha \ll 1$ , then with increasing of L, the absorption varies in the following way. For 0  $< L < l_s \eta$  is determined by expression (5) for the sharp boundary. For  $l_s < L < v_T / \omega$ , the absorption grows linearly with increasing L, so that  $\eta \approx 2\pi\omega(L+l_s)/c$  in these two ranges of scale lengths. For  $L \approx v_T / \omega$ , absorption reaches its maximum. Moreover, when  $\alpha > 1$ , the anomalous skin effect is replaced by the SIB (the integrodifferential equation for the field is reduced to an ordinary differential one). In this case,  $\eta$  decreases with increase of L according to Eq. (10). Thus, for the S-polarized electromagnetic wave in the regime of the anomalous skin effect, there is the maximum of absorption at the scales of inhomogeneity L of the order of magnitude of the scale of spatial dispersion  $v_T/\omega$ . If  $\alpha \ge 1$ , then at  $L \approx 0$  we are already in the regime of the highfrequency normal skin effect. In this case, in the range  $0 < L < c/\omega_p$ , absorption is determined by Eq. (5) for a sharp boundary, and at  $c/\omega_p < L$  by expression (10), i.e., it decreases with increasing L. If we optimize the absorption with respect to parameter  $\alpha$  and the scale length L, then it is obvious that the maximum will be at  $\alpha = 1$  and  $\omega L/v_T \approx 1$ .

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